

# A Novel Superstring in Four Dimensions and Grand Unification

B. B. Deo

*Physics Department, Utkal University, Bhubaneswar-751004, India.*

A string in four dimensions is constructed by supplementing it with forty four Majorana fermions. The central charge is 26. The fermions are grouped in such a way that the resulting action is supersymmetric. The energy momentum and current generators satisfy the super-Virasoro algebra. The tachyonic ground state decouples from the physical states. GSO projections are necessary for proving modular invariance. Space-time supersymmetry provides reasons to discard the tachyons and is substantiated for modes of zero mass. The symmetry group of the model descends to the low energy standard model group  $SU(3) \times SU_L(2) \times U_Y(1)$  through the Pati-Salam group. Left right symmetry is broken spontaneously and the mass of the tau neutrino is calculated to be about 1/25 electron volt.

PACS : 11.17.+y

## I. INTRODUCTION

String theory was invented [1] as a sequel to dual resonance models [2] to explain the properties of strongly interacting particles in four dimensions. Assuming a background gravitational field and demanding Weyl invariance, the Einstein equations of general relativity could be deduced. It was believed that about these classical solutions one can expand and find the quantum corrections. But difficulties arose at the quantum level. Eventhough the strong interaction amplitude obeyed crossing, it was no longer unitary. There were anomalies and ghosts. Due to these compelling reasons it was necessary for the open string to live in 26 dimensions [3,16]. At present the most successful theory is a ten dimensional superstring on a Calabi-Yau manifold or an orbifold. However, in order to realise the programme of the string unification of all the four types of interactions, one must eventually arrive at a theory in four flat space-time dimensions, with N=1 supersymmetry and chiral matter fields. This paper is an attempt in that direction.

A lot of research has been done to construct four dimensional strings [4], specially in the latter half of the eighties. Antoniadis et al [5] have constructed a four dimensional superstring supplemented by eighteen real fermions in trilinear coupling. The central charge of the construction is 15. Chang and Kumar [6] have discussed Thirring fermions, but again with the central charge at 15. Kawai et al [7] have also considered four dimensional models in a different context than the model proposed here. None of these models makes contact with the standard model.

In section II, we give the details of the supersymmetric model. Section III gives the usual quantization and super-Virasoro algebra is deduced in the section IV. Bosonic states are constructed in Section V. Fadeev- Popov ghosts are introduced and the BRST charge is explicitly given in section VI. Ramond states have been worked out in section VII. In section VIII, the mass spectrum of the model and the necessary GSO projections to eliminate the half integral spin states are introduced. In section IX, we show that these projections are necessary to prove the modular invariance of the model. Space-time supersymmetry algebra is satisfied and is shown to exist for the zero mass modes in section X. In section XI we show how the chain  $SO(44) \rightarrow SO(11) \rightarrow SO(6) \times SO(5) \rightarrow SU_C(3) \times SU_L(2) \times U_Y(1)$  is possible in this model. We calculate that the Pati-Salam group  $SU(4) \times SU_L(2) \times SU_R(2)$  breaks at an intermediate mass  $M_R \simeq 5 \times 10^{14}$  GeV giving the left-handed neutrino a small mass, which has now been observed in the top sector.

The literature on string theory is very vast and exist in most text books on the subject. The references serve only as a guide to elucidate the model.

## II. THE MODEL

The model essentially consists of 26 vector bosons of an open (closed) string in which there are the four bosonic coordinates of four dimensions and there are fortyfour Majorana fermions representing the remaining 22 bosonic coordinates [8]. We divide them into four groups . They are labelled by  $\mu = 0, 1, 2, 3$  and each group contains 11 fermions. These 11 fermions are again divided into two groups, one containing six and the other five. For convenience, in one group we have  $j = 1, 2, 3, 4, 5, 6$ , and in the other,  $k = 1, 2, 3, 4, 5$ .

The string action is

$$S = -\frac{1}{2\pi} \int d^2\sigma \left[ \partial_\alpha X^\mu \partial^\alpha X_\mu - i\bar{\psi}^{\mu,j} \rho^\alpha \partial_\alpha \psi_{\mu,j} - i\phi^{\mu,k} \rho^\alpha \partial_\alpha \phi_{\mu,k} \right] , \quad (1)$$

$\rho^\alpha$  are the two dimensional Dirac matrices

$$\rho^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} , \quad \rho^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \quad (2)$$

and obey

$$\{\rho^\alpha, \rho^\beta\} = -2\eta_{\alpha\beta} . \quad (3)$$

String co-ordinates  $X^\mu$  are scalars in  $(\sigma, \tau)$  space and vectors in target space. Similarly  $\psi^{\mu,j}$  are spinors in  $(\sigma, \tau)$  space and vectors in target space.

In general we follow the notations and conventions of reference [9] whenever omitted by us.  $X^\mu(\sigma, \tau)$  are the string coordinates. The Majorana fermions  $\psi$ 's and  $\phi$ 's are decomposed in the basis

$$\psi = \begin{pmatrix} \psi_- \\ \psi_+ \end{pmatrix} , \quad \text{and} \quad \phi = \begin{pmatrix} \phi_- \\ \phi_+ \end{pmatrix} . \quad (4)$$

The nonvanishing commutation and anticommutations are

$$[\dot{X}^\mu(\sigma, \tau), X^\nu(\sigma', \tau)] = -i\pi \delta(\sigma - \sigma') \eta^{\mu\nu} \quad (5)$$

$$\{\psi_A^\mu(\sigma, \tau), \psi_B^\nu(\sigma', \tau)\} = \pi \eta^{\mu\nu} \delta_{AB} \delta(\sigma - \sigma') \quad (6)$$

$$\{\phi_A^\mu(\sigma, \tau), \phi_B^\nu(\sigma', \tau)\} = \pi \eta^{\mu\nu} \delta_{AB} \delta(\sigma - \sigma') \quad (7)$$

The action is invariant under infinitesimal transformations

$$\delta X^\mu = \bar{\epsilon} \left( \sum_j \psi^{\mu,j} + i \sum_k \phi^{\mu,k} \right) \quad (8)$$

$$\delta \psi^{\mu,j} = -i \rho^\alpha \partial_\alpha X^\mu \epsilon \quad (9)$$

$$\delta \phi^{\mu,k} = +\rho^\alpha \partial_\alpha X^\mu \epsilon \quad (10)$$

where  $\epsilon$  is an infinitesimally constant anticommuting Majorana spinor. The commutator of the two supersymmetry transformations gives a spatial translation, namely

$$[\delta_1, \delta_2] X^\mu = a^\alpha \partial_\alpha X^\mu \quad (11)$$

and

$$[\delta_1, \delta_2] \Psi^\mu = a^\alpha \partial_\alpha \Psi^\mu \quad (12)$$

where

$$a^\alpha = 2i \bar{\epsilon}_1 \rho^\alpha \epsilon_2 \quad (13)$$

and

$$\Psi^\mu = \sum_j \psi^{\mu,j} + i \sum_k \phi^{\mu,k} \quad (14)$$

In deriving this, the Dirac equation for the spinors have been used. The Noether super-current is

$$J_\alpha = \frac{1}{2} \rho^\beta \rho_\alpha \Psi^\mu \partial_\beta X_\mu \quad (15)$$

We now follow the standard procedure. The light cone components of the current and energy momentum tensors are

$$J_+ = \partial_+ X_\mu \Psi_+^\mu \quad (16)$$

$$J_- = \partial_- X_\mu \Psi_-^\mu \quad (17)$$

$$T_{++} = \partial_+ X^\mu \partial_+ X_\mu + \frac{i}{2} \psi_+^{\mu, j} \partial_+ \psi_{+\mu, j} + \frac{i}{2} \phi_+^{\mu, k} \partial_+ \phi_{+\mu, k} \quad (18)$$

$$T_{--} = \partial_- X^\mu \partial_- X_\mu + \frac{i}{2} \psi_-^{\mu, j} \partial_- \psi_{-\mu, j} + \frac{i}{2} \phi_-^{\mu, k} \partial_- \phi_{-\mu, k} \quad (19)$$

where  $\partial_\pm = \frac{1}{2}(\partial_\tau \pm \partial_\sigma)$ .

To proceed further we note that in equation (8) and (14) we could have taken  $-i$  instead of  $+i$ . We now introduce a phase factor  $\eta_\phi$  to replace ‘ $i$ ’ in these equation.  $\eta_\phi$  depends on the number  $n_\phi$  of  $\phi_s$  (or its quanta), in a given individual term. Explicitly  $\eta_\phi = (-1)^{1/4 n_\phi (n_\phi - 1) + \frac{1}{2}}$ .  $\eta_\phi = i$  if  $n_\phi = 1$  reproducing ‘ $i$ ’ in the above equations. But  $\eta_\phi^2 = -1$  if  $n_\phi = 0$  where two  $\phi$ ’s have been contracted away and  $\eta_\phi^2 = 1$  if  $n_\phi = 2$ .

One now readily calculates the algebra

$$\begin{aligned} \{J_+(\sigma), J_+(\sigma')\} &= \pi \delta(\sigma - \sigma') T_{++}(\sigma) \\ \{J_-(\sigma), J_-(\sigma')\} &= \pi \delta(\sigma - \sigma') T_{--}(\sigma) \\ \{J_+(\sigma), J_-(\sigma')\} &= 0 \end{aligned} \quad (20)$$

The time like components of  $X^\mu$  are eliminated by the use of Virasoro constraints  $T_{++} = T_{--} = 0$ . In view of equation (20), we postulate that

$$0 = J_+ = J_- = T_{++} = T_{--} \quad (21)$$

$J_+$  is a sum of a real and imaginary term, The real term is a sum of six mutually independent  $\psi^{\mu, j}$ ’s and the imaginary term, the five mutually independent  $\phi^{\mu, k}$ ’s. It will be shown in Section V, that  $J_+ = 0$  constraint excludes all the eleven time like components of  $\psi$ ’s and  $\phi$ ’s from the physical space.

### III. QUANTIZATION

As usual the theory is quantized ( $\alpha_o^\mu = p^\mu$ ), with

$$X^\mu = x^\mu + p^\mu \tau + i \sum_{n \neq 0} \frac{1}{n} \alpha_n^\mu \exp^{-in\tau} \cos(n\sigma), \quad (22)$$

or

$$\partial_\pm X^\mu = \frac{1}{2} \sum_{n=-\infty}^{+\infty} \alpha_n^\mu e^{-in(\tau \pm \sigma)} \quad (23)$$

$$[\alpha_m^\mu, \alpha_n^\nu] = m \delta_{m+n} \eta^{\mu\nu} \quad (24)$$

While discussing the mass spectrum, it will be more illuminating to consider the closed string. The related additional quantas here and in wherever occurs will be denoted by attaching a tilde. For instance

$$\partial_- X_\mu^R = \sum_{n=-\infty}^{+\infty} \alpha_n^\mu e^{-2in(\sigma - \tau)} \quad (25)$$

$$\partial_+ X_\mu^L = \sum_{-\infty}^{+\infty} \tilde{\alpha}_n^\mu e^{-2in(\sigma+\tau)} \quad (26)$$

The transition formulas for closed strings can be easily effected. We consider the open string. We first choose the Neveu-Schwarz (NS) [10] boundary condition. Then the mode expansions of the fermions are

$$\psi_\pm^{\mu,j}(\sigma, \tau) = \frac{1}{\sqrt{2}} \sum_{r \in Z + \frac{1}{2}} b_r^{\mu,j} e^{-ir(\tau \pm \sigma)} \quad (27)$$

$$\phi_\pm^{\mu,k}(\sigma, \tau) = \frac{1}{\sqrt{2}} \sum_{r \in Z + \frac{1}{2}} b_r^{\mu,k} e^{-ir(\tau \pm \sigma)} \quad (28)$$

$$\Psi_\pm^{\mu,j}(\sigma, \tau) = \frac{1}{\sqrt{2}} \sum_{r \in Z + \frac{1}{2}} B_r e^{-ir(\tau \pm \sigma)} \quad (29)$$

The sum is over all the half-integer modes.

$$\{b_r^{\mu,j}, b_s^{\nu,j'}\} = \eta^{\mu\nu} \delta_{j,j'} \delta_{r+s} \quad (30)$$

$$\{b_r^{\mu,k}, b_s^{\nu,k'}\} = \eta^{\mu\nu} \delta_{k,k'} \delta_{r+s} \quad (31)$$

$$\{B_r^\mu, B_s^\nu\} = \eta^{\mu\nu} \delta_{r+s} . \quad (32)$$

#### IV. VIRASORO ALGEBRA

Virasoro generators [11] are given by the modes of the energy momentum tensor  $T_{++}$  and Noether current  $J_+$ ,

$$L_m^M = \frac{1}{\pi} \int_{-\pi}^{+\pi} d\sigma e^{im\sigma} T_{++} \quad (33)$$

$$G_r^M = \frac{\sqrt{2}}{\pi} \int_{-\pi}^{+\pi} d\sigma e^{ir\sigma} J_+ \quad (34)$$

‘ $M$ ’ stands for matter. In terms of creation and annihilation operators

$$L_m^M = L_m^{(\alpha)} + L_m^{(b)} + L_m^{(b')} \quad (35)$$

where

$$L_m^{(\alpha)} = \frac{1}{2} \sum_{n=-\infty}^{\infty} : \alpha_{-n} \cdot \alpha_{m+n} : \quad (36)$$

$$L_m^{(b)} = \frac{1}{2} \sum_{r=-\infty}^{\infty} (r + \frac{1}{2}m) : b_{-r} \cdot b_{m+r} : \quad (37)$$

$$L_m^{(b')} = \frac{1}{2} \sum_{r=-\infty}^{\infty} (r + \frac{1}{2}m) : b'_{-r} \cdot b'_{m+r} : \quad (38)$$

In each case normal ordering is required. The single dot implies the sum over all qualifying indices. The current generator is

$$G_r^M = \sum_{n=-\infty}^{\infty} \alpha_{-n} \cdot (b_{r+n} + \eta_\phi b'_{r+n}) = \sum_{n=-\infty}^{\infty} \alpha_{-n} \cdot (b_{r+n} + i b'_{r+n}) = \sum_{n=\infty} \alpha_n \cdot B_{r+n} \quad (39)$$

Following from eqn. (33) the Virasoro algebra is

$$[L_m^M, L_n^M] = (m - n) L_{m+n}^M + A(m) \delta_{m+n} \quad (40)$$

Using the relations

$$[L_m^M, \alpha_n^\mu] = -n \alpha_{n+m}^\mu \quad (41)$$

$$[L_m^M, B_n^\mu] = -(n + \frac{m}{2}) B_{n+m}^\mu \quad (42)$$

we get, also

$$[L_m^M, G_r^M] = \left( \frac{1}{2} m - r \right) G_{m+r}^M \quad (43)$$

The anticommutator  $\{G_r^M, G_s^M\}$  is obtained directly or by the use of the Jacobi identity

$$\{[G_r^M, G_s^M], L_m^M\} + \{[L_m^M, G_r^M], G_s^M\} + \{[L_m^M, G_s^M], G_r^M\} = 0 \quad (44)$$

which implies, consistent with equations (34) and (35),

$$\{G_r^M, G_s^M\} = 2L_{r+s}^M + B(r) \delta_{r+s} \quad (45)$$

$A(m)$  and  $B(r)$  are normal ordering anomalies. Taking the vacuum expectation value in the Fock ground state  $|0, 0\rangle$  with four momentum  $p^\mu = 0$  of the commutator  $[L_1, L_{-1}]$  and  $[L_2, L_{-2}]$ , it is easily found that

$$A(m) = \frac{26}{12}(m^3 - m) = \frac{C}{12}(m^3 - m) \quad (46)$$

and using the Jacobi identity

$$B(r) = \frac{A(2r)}{2r} \quad (47a)$$

$$B(r) = \frac{26}{3} \left( r^2 - \frac{1}{4} \right) = \frac{C}{3} \left( r^2 - \frac{1}{4} \right) \quad (47b)$$

The central charge  $C = 26$ . This is what is expected. Each bosonic coordinate contribute 1 and each fermionic ones contribute 1/2, so that the total central charge is +26.

For closed strings there will be another set of tilded generators satisfying the same algebra.

## V. BOSONIC STATES

A physical bosonic state  $\Phi$  which should be invariant under  $SO(6) \times SO(5)$  internal symmetry group and can be conveniently constructed by operating the generators  $L$ 's and  $G$ 's on the vacuum. They satisfy

$$L_m^M | \Phi \rangle = 0 \quad m > 0 \quad (48)$$

$$G_r^M | \Phi \rangle = 0 \quad r > 0 \quad (49)$$

These conditions enable to exclude the time like quanta from the physical spectrum. Specialising to a rest frame we write the conditions (48) as

$$\frac{1}{2}p^0\alpha_m^0|\Phi\rangle + (\text{terms quadratic in oscillators})|\Phi\rangle = 0 \quad (50)$$

In this frame, the physical states are generated effectively by the space components of the oscillators only; so that  $\alpha_m^0|\Phi\rangle = 0$  following from the constraint that the energy momentum tensor vanishes. Using the condition (49),

$$[G_r^M, \alpha_m^0]|\Phi\rangle = mb_{m+r}^0|\Phi\rangle = 0$$

means

$$(b_r^{0,1} + \dots + b_r^{0,6})|\Phi\rangle = 0 \quad (51)$$

$$(b_r^{0,1} + \dots + b_r^{0,5})|\Phi\rangle = 0 \quad (52)$$

$b_r^{0,1}$  to  $b_r^{0,6}$  or  $b_r^{0,1}$  to  $b_r^{0,5}$  are all independent annihilation operators for  $r > 0$  and there is no relation between them. Therefore  $\Phi$  decouple from the eleven time like components  $b_r^j$ 's or  $b_r^k$ 's, for, otherwise the equality to zero in equations (51) and (52) cannot be achieved. Thus the vanishing of the energy-momentum tensor and the current excludes all the time like components from the physical space. No negative norm state will show up in the physical spectrum and at the same time preserve  $SO(6) \times SO(5)$  internal symmetry. The above arguments are only qualitative.

Let us make a detailed investigation to ensure that there are no negative norm physical states. We shall do this by constructing the zero norm states or the 'null' physical states. Due to the GSO condition, which we shall study later, the physical states will be obtained by operation of the product of even number of  $G$ 's. So the lowest state above the tachyonic state is

$$|\Psi\rangle = L_{-1}|\chi_1\rangle + G_{-1/2}G_{-1/2}|\chi_2\rangle$$

But  $G_{-1/2}G_{-1/2} = \frac{1}{2}\{G_{-1/2}, G_{-1/2}\} = L_{-1}$ . Without loss of generality, the state is

$$|\Psi\rangle = L_{-1}|\tilde{\chi}\rangle \quad (53)$$

This state to be physical, it must satisfy  $(L_0 - 1)|\Psi\rangle = 0$  which is true if  $L_0|\tilde{\chi}\rangle = 0$ . The norm  $\langle\Psi|\Psi\rangle = \langle\tilde{\chi}|L_1L_{-1}|\tilde{\chi}\rangle = 2\langle\tilde{\chi}|L_0|\tilde{\chi}\rangle = 0$ . Let us consider the next higher mass state

$$|\Psi\rangle = L_{-2}|\chi_1\rangle + L_{-1}^2|\chi_2\rangle + (G_{-3/2}G_{-1/2} + \lambda G_{-1/2}G_{-3/2})|\chi_3\rangle + G_{-1/2}G_{-1/2}G_{-1/2}G_{-1/2}|\chi_4\rangle + \dots$$

It can be shown that  $G_{-3/2}G_{-1/2}|\tilde{\chi}\rangle = (\beta_1 L_{-1}^2 + \beta_2 L_{-2})|\tilde{\chi}\rangle$ . The coefficients  $\beta_1$  and  $\beta_2$  can be calculated by evaluating  $[L_1, G_{-3/2}G_{-1/2}]|\tilde{\chi}\rangle$  and  $[L_2, G_{-3/2}G_{-1/2}]|\tilde{\chi}\rangle$ .  $G_{-1/2}^4$  is proportional to  $L_{-1}^2$ . So, in essence, we have the next excited state as

$$|\Psi\rangle = (L_{-2} + \gamma L_{-1}^2)|\tilde{\chi}\rangle \quad (54)$$

The condition  $(L_0 - 1)|\Psi\rangle = 0$  is satisfied if  $(L_0 + 1)|\tilde{\chi}\rangle = 0$ . Further the physical state condition  $L_1|\Psi\rangle = 0$  gives the value of  $\gamma = 3/2$ . The norm is easily obtained as

$$\langle\Psi|\Psi\rangle = \frac{1}{2}(C - 26) \quad (55)$$

This is negative for  $C < 26$  and vanishes for  $C = 26$ . So the critical central charge is 26. It is easily checked that  $L_2|\Psi\rangle$  also vanishes for  $C = 26$ .

To find the role of  $b$  and  $b'$  modes, let us calculate the norm of the following state with  $p^2 = 2$

$$(L_{-2} + 3/2 L_{-1}^2)|0, p\rangle = \left(L_{-2}^{(\alpha)} + \frac{3}{2}L_{-1}^{(\alpha)^2}\right)|0, p\rangle + \left(L_{-2}^{(b)} + \frac{3}{2}L_{-1}^{(b)^2}\right)|0, p\rangle + \left(L_{-2}^{(b')} + \frac{3}{2}L_{-1}^{(b')^2}\right)|0, p\rangle \quad (56)$$

The norm of the first term is equal to  $-11$  as calculated in reference [9].

Noting that  $L_{-1}^{(b)}|0, p\rangle = L_{-1}^{(b')}|0, p\rangle = 0$ ;  $L_{-2}^{(b)} = \frac{1}{2}b_{-3/2} \cdot b_{-1/2}$  and  $L_{-2}^{(b')} = \frac{1}{2}b'_{-3/2} \cdot b'_{-1/2}$  the norms of the second and third terms are  $\frac{1}{4}(\delta_{\mu\mu}\delta_{jj}) = 6$  and  $\frac{1}{4}(\delta_{\mu\mu}\delta_{kk}) = 5$  respectively. The norm of the state given in equation (56) is  $-11 + 6 + 5 = 0$

Since  $L_1 = G_{1/2}^2$ ,  $L_1|\Psi\rangle = 0$  implies  $G_{1/2}|\Psi\rangle = 0$ .  $G_{3/2}$  can be expressed as a commutator of  $L_1$  and  $G_{1/2}$ , so that  $G_{3/2}|\Psi\rangle = 0$ . Further  $L_2|\Psi\rangle = \frac{1}{2}\{G_{3/2}, G_{1/2}\}|\Psi\rangle = 0$  and so on, satisfying all the physical state conditions.

## VI. GHOSTS

For obtaining a zero central charge so that the anomalies cancel out and natural ghosts are isolated, Faddeev-Popov (FP) ghosts [12] are introduced. The FP ghost action is

$$S_{FP} = \frac{1}{\pi} \int (c^+ \partial_- b_{++} + c^- \partial_+ b_{--}) d^2 \sigma \quad (57)$$

where the ghost fields  $b$  and  $c$  satisfy the anticommutator relations

$$\{b_{++}(\sigma, \tau), c^+(\sigma', \tau)\} = 2\pi \delta(\sigma - \sigma') \quad (58)$$

$$\{b_{--}(\sigma, \tau), c^-(\sigma', \tau)\} = 2\pi \delta(\sigma - \sigma') \quad (59)$$

and are quantized with the mode expansions

$$c^\pm = \sum_{-\infty}^{\infty} c_n e^{-in(\tau \pm \sigma)} \quad (60)$$

$$b_{\pm\pm} = \sum_{-\infty}^{\infty} b_n e^{-in(\tau \pm \sigma)} \quad (61)$$

The canonical anticommutator relations for  $c_n$ 's and  $b_n$ 's are

$$\{c_m, b_n\} = \delta_{m+n} \quad (62)$$

$$\{c_m, c_n\} = \{b_m, b_n\} = 0 \quad (63)$$

Deriving the energy momentum tensor from the action and making the mode expansion, the Virasoro generators for the ghosts (G) are

$$L_m^G = \sum_{n=-\infty}^{\infty} (m-n) b_{m+n} c_{-n} - a \delta_m \quad (64)$$

where  $a$  is the normal ordering constant. These generators satisfy the algebra

$$[L_m^G, L_n^G] = (m-n) L_{m+n}^G + A^G(m) \delta_{m+n} \quad (65)$$

The anomaly term is deduced as before and give

$$A^G(m) = \frac{1}{6}(m - 13m^3) + 2a m \quad (66)$$

With  $a = 1$ , this anomaly term becomes

$$A^G(m) = -\frac{26}{12}(m^3 - m) \quad (67)$$

$$B^G(r) = -\frac{26}{3} \left( r^2 - \frac{1}{4} \right) \quad (68)$$

The central charge is  $-26$  and cancels the normal ordder  $A(m)$  and  $B(r)$  of the  $L$  and  $G$  generators. Noting that

$$[L_m^G, c_n] = -(2m+n)c_{n+m} \quad (69)$$

it is possible to construct an equation for the generator for the current of the ghost sector,

$$G_r^{gh} = \sum_p \left(\frac{p}{2} - r\right) c_{-p} G_{p+r}^{gh} \quad (70)$$

so that

$$[L_m^G, G_r^{gh}] = (m/2 - r) G_{m+r}^{gh} \quad (71)$$

From Jacobi identity (65)

$$\{G_r^{gh}, G_s^{gh}\} = 2L_{r+s}^G + \delta_{r+s} B^G(r) \quad (72)$$

It immediately follows that

$$G_r^{gh^2} = L_{2r}^G \quad (73)$$

Since  $L_{2r}^G$  is well defined, equation (70) has a nonvanishing solution for  $G_r^{gh}$ . In practice, the products of even number of  $G_r^{gh}$ 's occur in calculations and they can be evaluated in terms of  $L_{2r}^G$ 's.

The total current generator is

$$G_r = G_r^M + G_r^{gh} \quad (74)$$

thus we have the anomaly free Super Virasoro algebra,

$$[L_m, L_n] = (m - n) L_{m+n} \quad (75)$$

$$[L_m, G_r] = (m/2 - r) G_{r+m} \quad (76)$$

$$[G_r, G_s] = 2L_{r+s} \quad (77)$$

Thus from the usual conformal field theory we have obtained the algebra of a superconformal field theory. This is the novelty of the present formulation. The BRST [13] charge operator is

$$Q_{BRST} = \sum_{-\infty}^{\infty} L_{-m}^M c_m - \frac{1}{2} \sum_{-\infty}^{\infty} (m - n) : c_{-m} c_{-n} b_{m+n} : - a c_0 \quad (78)$$

and is nilpotent for  $a = 1$ . The physical states are such that  $Q_{BRST} |phys\rangle = 0$ .

## VII. FERMIONIC STATES

The above deductions can be repeated for Ramond sector [14]. We write the main equations. The mode expansion for the fermions are

$$\psi_{\pm}^{\mu,j}(\sigma, \tau) = \frac{1}{\sqrt{2}} \sum_{-\infty}^{\infty} d_m^{\mu,j} e^{-im(\tau \pm \sigma)} \quad (79)$$

$$\phi_{\pm}^{\mu,j}(\sigma, \tau) = \frac{1}{\sqrt{2}} \sum_{-\infty}^{\infty} d_m^{\prime\mu,j} e^{-im(\tau \pm \sigma)} \quad (80)$$

The generators of the Virasoro operators are

$$L_m^M = L_m^{(\alpha)} + L_m^{(d)} + L_m^{(d')} \quad (81)$$

$$L_m^{(d)} = \frac{1}{2} \sum_{n=-\infty}^{\infty} \left(n + \frac{1}{2}m\right) : d_{-n} \cdot d_{m+n} : \quad (82)$$



$$L_m^{(d')} = \frac{1}{2} \sum_{n=-\infty}^{\infty} (n + \frac{1}{2}m) : d'_{-n} \cdot d'_{m+n} : \quad (83)$$

and the fermionic current generator is

$$F_m^M = \sum_{n=-\infty}^{\infty} \alpha_{-n} \cdot (d_{n+m} + i d'_{n+m}) = \sum_{n=-\infty}^{\infty} \alpha_{-n} \cdot D_{n+m} \quad (84)$$

The Ramond sector Virasoro algebra is the same as the NS-sector with the replacement of G's by F's. It is necessary to define  $L_o$  suitably to keep the anomaly equations the same [9].

In this Ramond sector, a physical state  $|\Phi\rangle$  should satisfy

$$F_n |\Phi\rangle = L_n |\Phi\rangle = 0 \quad \text{for } n > 0 \quad (85)$$

The normal order anomaly constant in the anticommutables of the Ramond current generators has to be evaluated with care, because the definition of  $F_0$  does not have a normal ordering ambiguity. So  $F_0^2 = L_0$ . Using commutation relation (43) with  $G$  replaced by  $F$  and the Jacobi Identity we get

$$\{F_r, F_{-r}\} = \frac{2}{r} \{[L_r, F_0], F_{-r}\} = 2L_0 + \frac{4}{r} A(r)$$

So

$$B(r) = \frac{4}{r} A(r) \quad (86)$$

$$B(r) = \frac{C}{3}(r^2 - 1), \quad r \neq 0 \quad (87)$$

A physical state in the fermionic sector satisfies

$$(L_0 - 1) |\Psi\rangle = 0 \quad (88)$$

It follows that

$$(F_0^2 - 1) |\Psi\rangle = (F_0 - 1)(F_0 + 1) |\Psi\rangle = 0$$

The Ramond fermionic vacuum is also tachyonic and could have been the supersymmetric partner of the bosonic N - S, tachyonic vacuum. They are eliminated from Fock spaces by space time supersymmetry.

The construction of 'null' physical states becomes much simpler because all  $F_{-m}$  terms can be assigned to  $L_{-m}$  terms by the commutation relation  $F_{-m} = 2[L_0, L_{-m}]/m$  and  $F_0$  has eigen values which are roots of eigen values of  $L_0$  acting on the generic states or states constructed out of the generic states. Thus the zero mass null physical state with  $L_0 |\tilde{\chi}\rangle = F_0^2 |\tilde{\chi}\rangle = 0$  is simply

$$|\Psi\rangle = L_{-1} |\tilde{\chi}\rangle \quad (89)$$

with  $L_1 |\Psi\rangle = F_1 |\Psi\rangle = 0$ . The next excited state with  $(L_0 + 1) |\tilde{\chi}\rangle$  becomes the same as in the bosonic sector. Obtained from the condition  $L_1 |\Psi\rangle = 0$ ,

$$|\Psi\rangle = (L_{-2} + \frac{3}{2} L_{-1}^2) |\tilde{\chi}\rangle$$

The norm  $\langle\Psi | \Psi\rangle = (C - 26)/2$  and vanishes for  $C = 26$ . It is easy to check that all physical state conditions are satisfied.  $F_1 |\Psi\rangle = 2 [L_1, F_0] |\Psi\rangle = 0$  since  $L_1 |\Psi\rangle = 0$  and  $F_0 |\Psi\rangle = |\Psi\rangle$ ,  $L_2 |\Psi\rangle = F_1 F_1 |\Psi\rangle = 0$  and  $F_2 |\Psi\rangle = [L_2, F_0] |\Psi\rangle = 0$ . For  $C = 26$ , there are no negative norm states in the Ramond sector as well.

The ghost current in the Ramond sector satisfies the equation

$$F_m^{gh} = \sum_p (\frac{p}{2} - m) c_{-p} F_{m+p}^{gh} \quad (90)$$

so that

$$[L_m^G, F_n^{gh}] = \left(\frac{m}{2} - n\right) F_{m+n}^{gh} \quad (91)$$

we can construct  $F_0^{gh}$  with the help of an anti commuting object  $\Gamma_n$  which satisfy

$$\{\Gamma_n, \Gamma_m\} = 2\delta_{m,n} \quad (92)$$

It is important to write  $L_0^G$  in terms of positive integrals as

$$L_0^G = \sum_{n=1}^{\infty} n(b_{-n}c_n + c_{-n}b_n) \quad (93)$$

It is found that

$$F_0^{gh} = \sum_{n=1}^{\infty} \sqrt{n}\Gamma_n(b_{-n}c_n + c_{-n}b_n) \quad (94)$$

All other F's can be constructed by the use of the equations of super Virasoro algebra.

From equation (67) and (86), the ghost current anomaly constant is  $B^G(r) = -\frac{26}{3}(r^2 - 1)$  and cancels out the  $B(r)$  of equation(87). The total current anomalies in both the sectors vanish.

### VIII. THE MASS SPECTRUM

The ghosts are not coupled to the physical states. Therefore the latter must be of the form (up to null state) [15].

$$|\{n\} p\rangle_M \otimes c_1|0\rangle_G \quad (95)$$

$|\{n\} p\rangle_M$  denotes the occupation numbers and momentum of the physical matter states. The operator  $c_1$  lowers the energy of the state by one unit and is necessary for BRST invariance. The ghost excitation is responsible for lowering the ground state energy which produces the tachyon.

$$(L_0^M - 1) |phys\rangle = 0 \quad (96)$$

Therefore, the mass shell condition is

$$\alpha' M^2 = N^B + N^F - 1 \quad (97)$$

where

$$N^B = \sum_{m=1}^{\infty} \alpha_{-m} \alpha_m \quad (98)$$

or

$$N^F = \sum_{r=1/2}^{\infty} r (b_{-r} \cdot b_r + b'_{-r} \cdot b'_r) \quad (NS). \quad (99)$$

Due to the presence of Ramond and Neveu-Schwartz sectors with periodic and anti-periodic boundary conditions, we can effect a GSO projection [16] on the mass spectrum on the NSR model [17]. Here the projection should refer to the unprimed and the primed quantas separately. The desired projection is

$$G = \frac{1}{4}(1 + (-1)^F)(1 + (-1)^{F'}) \quad (100)$$

where  $F = \sum b_{-r} \cdot b_r$ ;  $F' = \sum b'_{-r} \cdot b'_r$ . This will eliminate the half integral values from the mass spectrum by choosing  $G=1$ .

For closed strings we have a similar separation as in Eq. (95), namely the left-handed states will be in the form

$$|\{\tilde{n}\} p\rangle_M \otimes \tilde{c}_1|0\rangle_G \quad (101)$$

The mass spectrum can be written as

$$\frac{1}{2}\alpha' M^2 = N + \tilde{N} - 1 - \tilde{1} \quad (102)$$

## IX. MODULAR INVARIANCE

The GSO projection is necessary for the modular invariance of the theory. We follow the notation of Seiberg and Witten [18]. Following Kaku [17], the spin structure  $\chi(--, \tau)$  for a single fermion is given by

$$\chi(--, \tau) = q^{-1/24} \text{Tr } q^2 \sum_n n \psi_{-n} \psi_n = q^{-1/24} \prod_{n=1}^{\infty} (1 + q^{2n-1}) = \sqrt{\frac{\Theta_3(\tau)}{\eta(\tau)}}, \quad (103)$$

where  $\Theta$ 's will be the Jacobi Theta functions  $\Theta(\theta, \tau)$  [18],  $q = e^{i\pi\tau}$  and  $\eta(\tau)(2\pi) = \Theta_1^{1/3}(\tau)$ . The path integral functions of Seiberg and Witten for the twenty four unprimed oscillators are

$$A(--, \tau) = (\Theta_3(\tau)/\eta(\tau))^{12}, \quad (104)$$

This is normalised to one. similarly

$$A((+-), \tau) = A(--, \frac{\tau}{1+\tau}) = -(\Theta_2(\tau)/\eta(\tau))^{12}, \quad (105)$$

and

$$A((-+), \tau) = A(+-, -\frac{1}{\tau}) = -(\Theta_4(\tau)/\eta(\tau))^{12}, \quad (106)$$

$$A(++), \tau) = 0. \quad (107)$$

It is easily checked that the sum

$$A(\tau) = (\Theta_3(\tau)/\eta(\tau))^{12} - (\Theta_2(\tau)/\eta(\tau))^{12} - (\Theta_4(\tau)/\eta(\tau))^{12} \quad (108)$$

is modular invariant, using the properties of the theta functions given in [18].

For the twenty primed oscillators it is not so straightforward because of the ambiguity of fractional powers of unity. If we prescribe a normalization  $1 = 1^{1/2} = \sqrt{e^{2i\pi}}$ , then

$$A'(--, \tau) = (\Theta(\tau)/\eta(\tau))^{10}, \quad (109)$$

$$A'((+-), \tau) = \sqrt{e^{i\pi}} (\Theta_2(\tau)/\eta(\tau))^{10}, \quad (110)$$

$$A'((-+), \tau) = \sqrt{e^{i\pi}} (\Theta_4(\tau)/\eta(\tau))^{10}, \quad (111)$$

$$A'(++), \tau) = 0. \quad (112)$$

The sum  $(\Theta_3^{10}(\tau) + \sqrt{e^{i\pi}}\Theta_2^{10}(\tau) + \sqrt{e^{i\pi}}\Theta_4^{10}(\tau))/\eta^{10}(\tau)$  is also modular invariant upto the factor of cube root and fractional roots of unity. The sum of the moduli is, of course, modular invariant. It is easy to construct the modular invariant partition function for the two physical bosons, namely

$$\mathcal{P}_B(\tau) = (Im \tau)^{-2} \Delta^{-2}(\tau) \bar{\Delta}^{-2}(\tau), \quad (113)$$

in four dimensions [20]. For the normal ordering constant  $-1$  in  $q^{2(L_0-1)}$ ,  $-2/12$  comes from the bosons and  $-44/24$  comes from the fermions adding to  $-1$  for both the NS and the R sectors.

## X. SPACE-TIME SUPERSYMMETRY

So far the drawback of the model, is the existence of the tachyonic vacuum in both the bosonic and the fermionic sectors. One should examine further restrictions imposed on the Fock space due to the space time supersymmetric algebra. It is already been noted in reference [22], that a standard like model  $SU(3) \times SU(2) \times U(1) \times U(1)$  can be space time supersymmetric. The supersymmetric charge  $Q$  should be such that

$$\delta X^\mu = [X^\mu, \bar{Q} \cdot \epsilon] = \bar{\epsilon} \cdot \psi^\mu \quad (114)$$

and

$$\delta \psi^\mu = [\psi^\mu, \bar{Q} \cdot \epsilon] = -i \rho^\alpha \partial_\alpha X^\mu \epsilon \quad (115)$$

A simple inspection shows that

$$\bar{Q} = -\frac{i}{\pi} \int_0^\pi d\sigma \psi_\mu \rho^\alpha \rho^0 \partial_\alpha X^\mu \quad (116)$$

leading to

$$Q^\dagger = -\frac{i}{\pi} \int_0^\pi d\sigma \psi_\mu \rho^\alpha \rho^0 \partial_\alpha X^\mu \quad (117)$$

and

$$Q = \frac{i}{\pi} \int_0^\pi \rho^0 \rho^{\alpha\dagger} \partial_\alpha X^\mu \psi_\mu d\sigma \quad (118)$$

By a somewhat lengthy calculation it is deduced that

$$\sum_\alpha \{Q_\alpha^\dagger, Q_\alpha\} = 2H \quad (119)$$

where  $H$  is the Hamiltonian of the system. It follows that for the ground state  $|\Phi_0\rangle$  in the Fock space

$$\sum_\alpha |Q_\alpha |\phi_0\rangle|^2 = 2\langle\phi_0|H|\phi_0\rangle \geq 0 \quad (120)$$

It is essential that the tachyonic vacuum should be discarded from the physical Fock space and relegated to the ghost space to satisfy this result of the exact space-time supersymmetry. This has to be done in addition to the GSO projection. The ground state is massless.

Admissible  $SO(6) \times SO(5)$  symmetric Fock space states are

$$NS \text{ eigenstates : } \prod_{n,\mu} \prod_{m,\nu} \{\alpha_{-n}^\mu\} \{B_{-m}^\nu\} |0\rangle$$

$$R \text{ eigenstates : } \prod_{n,\mu} \prod_{m,0} \{\alpha_{-n}^\mu\} \{D_{-m}^\nu\} |0\rangle_u$$

Both the tachyons must be omitted and GSO projection is implied for the N S eigenstates. Let us construct the zero mass modes. The tachyonic vacuum will be denoted by  $|0\rangle$  and the zero mass ground state by  $|\phi_0\rangle$ . We start with the supergravity multiplet. The ground state

$$B_{-1/2}^\mu B_{-1/2}^\nu |0\rangle_{\epsilon_{\mu\nu}} \quad (121)$$

has zero mass. Due to the physical state conditions  $G_{1/2} |\phi_0\rangle = 0$

$$p^\mu \epsilon_{\mu\nu} = p^\nu \epsilon_{\mu\nu} = 0 \quad (122)$$

It describes a massless antisymmetric tensor  $A_{\mu\nu} = 1/2(\epsilon_{\mu\nu} - \epsilon_{\nu\mu})$ , which turns out to be a pseudoscalar, a massless scalar  $\epsilon_{\mu\mu}$  of spin 0 and a massless symmetric terms of spin 2:  $\epsilon_{\mu\nu} + 1/2(\epsilon_{\mu\nu} + \epsilon_{\nu\mu})$ , which is traceless.

The other zero mass spinorial states are

$$\alpha_{-1}^{\mu} | 0 \rangle u_{1\mu} \quad (123)$$

$$D_{-1}^{\mu} | 0 \rangle u_{2\mu} \quad (124)$$

$u_{1\mu}, u_{2\mu}$  are spinor four vectors and are distinguished by [9],

$$\gamma_5 u_{1\mu} = u_{1\mu} \quad (125)$$

$$\gamma_S u_{2\mu} = -u_{2\mu} \quad (126)$$

We shall consider them together as a four component spin vector  $u_{\mu}$ . The condition  $F_0 | \phi_0 \rangle = 0$ ,  $F_1 | \phi_0 \rangle = 0$ ,  $L_1 | \phi_0 \rangle = 0$  lead to the condition

$$\gamma \cdot p u_{\mu} = p^{\mu} u_{\mu} = \gamma^{\mu} u_{\mu} = 0 \quad (127)$$

This state contains not only a spin 3/2 but also a spin 1/2 state. They can be projected out. The details have been given by GSO in reference [16].

We now count the number of physical degrees of freedom [16]

Graviton	2 degrees of freedom:	$\rho_{\mu}^a$
Dilaton, $\epsilon_{\mu\mu}$ ,	1	A
Antisymmetric tensor	1	B
Spin 3/2	2	$u_{\mu}$
spin 1/2	2	u

The numbers of the fermions and the bosons are equal. They can be grouped together as the gravitational ( $\rho_{\mu}^a, u_{\mu}$ ) and the matter (A, B, u) multiplets.

The massless ground state vector is represented by

$$\alpha_{-1}^{\mu} | 0 \rangle \epsilon_{\mu}(p) \quad (128)$$

Here, because of the  $L_0$  condition,  $p^2 \epsilon_{\mu} = 0$ : The constraint  $L_1 | \phi_0 \rangle = 0$  gives the Lovertz condition  $p \cdot \epsilon = 0$ . The external photon polarisation vector can be subjected to an on shell gauge transformation  $\epsilon_{\mu}(p) \rightarrow \epsilon_{\mu}(p) + \lambda p_{\mu}$ . Therefore the state

$$p_{\mu} \alpha_{-1}^{\mu} | 0 \rangle \lambda = L_{-1} | 0 \rangle \lambda \quad (129)$$

decouples from the physical system. There are only two degrees of freedom left. However, from the Ramond sector we have the spinor (gaugino)

$$p_{\mu} \alpha_{-1}^{\mu} | 0 \rangle u(p) = F_{-1} | 0 \rangle u(p) \quad (130)$$

with  $\gamma \cdot p u(p) = 0$  from the physical state condition. Further, as already noted,  $\gamma_5 u(p) = u(p)$ . So the member of the fermionic degrees of freedom is again two, just like the vector boson. Thus for all the zero mass states the bosonic and the fermionic degrees of freedom are equal.

## XI. APPROACH TO STANDARD MODEL

One of the main motivation of constructing this superstring is to show that the internal symmetry group makes a direct contact with the standard model which explains all available experimental data with a high degree of accuracy. Since there are forty four fermions, the internal symmetry group was  $SO(44)$ . We divided these fermions in groups of eleven where each group was characterised by a space-time index  $\mu = 0, 1, 2, 3$ . All the four groups are similar, but not identical. The states when acted upon by creation/annihilation operators with  $\mu = 0$  are eliminated due to Virasoro constraints and the states with negative norm are absent. The other three groups of eleven,  $\mu = 1, 2, 3$  are identical and have  $SO(11)$  symmetry. These may be construed to be the three generations of the standard model.

By partitioning further, each  $SO(11)$  has been broken up into a product of  $SO(6)$  and  $SO(5)$ . so the internal symmetry of the model is  $SU(4) \times SO(5)$ . According to Slansky [21],  $SO(5)$  can break to  $SU(2) \times SU(2)$ . Thus we are led to the Pati-Salam group [23],  $SU(4) \times SU_L(2) \times SU_R(2)$ . The most convenient scheme of descending to the standard model is

$$\begin{aligned}
SO(11) &\longrightarrow SO(6) \times SO(5) \\
&\downarrow M_x \\
&SU(4) \times SU_L(2) \times SU_R(2) \\
&\downarrow M_R \\
&\times \\
&\downarrow M_S \\
&SU_C(3) \times SU_L(2) \times U_Y(1)
\end{aligned} \tag{131}$$

Such a scheme and similar ones have been extensively studied [24]. Invoking charge quantisation,  $SU(4)$  may be broken to  $U_{B-L}(1) \times SU_C(3)$  and subsequently  $U_{B-L}(1)$  may squeeze with  $SU_R(2)$  to yield  $SU_Y(1)$ . Unification mass is  $M_X = M_{GUT}$ , the left-right symmetry breaks at  $M_R$  and supersymmetry is broken at  $M_S = M_{SUSY}$ . The renormalisation equations for the evolution of the coupling constants are easily written down [25].

We denote  $\alpha_i = g_i^2/4\pi$  where  $g_i$  is the constant related to the  $i^{th}$  group,  $\alpha_G = g_u^2/4\pi$  where  $g_u$  is the coupling constant at the GUT energy and  $t_{XY} = \frac{1}{2\pi} \log_e M_X/M_Y$ . The lowest order evolution equations are

$$\alpha_3^{-1}(M_Z) = \alpha_G^{-1} + b_3 t_{SZ} + b_{3s} t_{RS} + b_{4s} t_{XR}, \tag{132}$$

$$\alpha_2^{-1}(M_Z) = \alpha_G^{-1} + b_2 t_{SZ} + b_{2s} t_{RS} + b_{2s} t_{XR}, \tag{133}$$

and

$$\alpha_1^{-1}(M_Z) = \alpha_G^{-1} + b_1 t_{SZ} + b_{1s} t_{RS} + \left(\frac{2}{5} b_{4s} + \frac{3}{5} b_{2s}\right) t_{XR}. \tag{134}$$

$b_i$  and  $b_{is}$  are the well known non susy and susy coefficients of the  $\beta$ -function respectively. The experimental values at  $M_Z = 91.18$  GeV are calculated to be [26]

$$\alpha_1^{-1} = 59.036, \alpha_2^{-1} = 29.656, \alpha_3^{-1} = 7.69 \tag{135}$$

To these, we add the expected string unification value

$$M_X = M_{GUT} = M_{string} = g_U(5 \times 10^{17}) GeV \tag{136}$$

We have four unknown quantities to calculate from the four known values, equations (135) and (136).

Notice that the quantities,  $b_1 - 3/5 b_2 = 6$ ,  $b_{1s} - 3/5 b_{2s} = 6$ ,  $b_3 = -7$ ,  $b_{3s} = -3$  and  $b_{4s} = -6$  are independent of the required number of Higgs doublets. so we rewrite the above three equations as

$$\alpha_1^{-1} - 3/5 \alpha_2^{-1} - 2/5 \alpha_3^{-1} = 8.8 t_{SZ} + 7.2 t_{RS} \tag{137}$$

$$\alpha_1^{-1} - 3/5 \alpha_2^{-1} = 2/5 \alpha_G^{-1} + 6 t_{SZ} + 6 t_{RS} - 2.4 t_{XR} \tag{138}$$

$$\alpha_3^{-1} = \alpha_G^{-1} - 7 t_{SZ} - 3 t_{RS} - 6 t_{XR} \tag{139}$$

The solutions are  $M_{SUSY} = 5 \times 10^9$  GeV,  $M_R = 5 \times 10^{14}$  GeV,  $M_X = 2.87 \times 10^{17}$  and  $g_u = 0.566$ . With the value of  $M_R$  found here, the mass of the left-handed tau neutrino [28] is calculated to be about  $1/25 eV$ . following references [24] and [27]. We have used  $m_{top}(M_R) \cong 140 GeV$  in the formula for the neutrino mass  $m_{\nu\tau}$ ,

$$m_{\nu\tau} = -\frac{m_{top}^2}{M_R} \tag{140}$$

This is a very important result of the model.

## XII. CONCLUSION

It is remarkable that we have been able to discuss physics from the Planck scale to the Kamiokanda neutrino scale within the same framework. The starting point has been a Nambu-Gatto string in four dimensions to which forty four Majorana neutrinos in groups of four have been added. The resulting string has an action which is supersymmetric. Super-Virasoro algebra for the energymomentum tensor and current generators is established. Conformal ghosts are introduced whose contributions cancel the anomalies. BRST charge is explicitly constructed.

The main drawback of the theory is the presence of the two tachyons in the bosonic and fermionic sector even after GSO projections. Since the space-time supersymmetry algebra is satisfied by the action, the two tachyons must be discarded from the physical spectrum.

The internal symmetry of the string is  $SO(6) \times SO(5)$  which breaks to the Pati-Salam group  $SU(4) \times SU_L(2) \times SU_R(2)$  at the string scale. The left right symmetry and supersymmetry are broken at intermediate mass scales. By the usual see-saw mechanism, the left handed neutrino develops a small mass of about  $\frac{1}{25}$  ev. Finally the descent is complete at  $SU_C(3) \times SU_L(2) \times U_Y(1)$ . There is no gap left between  $M_{GUT}$  and  $M_{string}$  by choice.

### Acknowledgement

I have profited from discussions with Dr. J. Maharana and Dr. S. Mahapatra. I thank Sri D. Pradhan for computer compilation and the Institute of Physics for providing Library and Computer facilities.

- [1] Y. Nambu, in *Symmetries and quark models*, ed. R. Chand (Gordon and Breach), (1970); H. B. Niclson, 15<sup>th</sup> International Conference on High Energy Physics, Kiev (1970); L. Susskind, Phys. Rev. **D1**, 1182 (1970).
- [2] R. Dolen, D. Horn and C. Schmid, Phys. Rev. Lett. **19**, 402 (1967); Phys. Rev. **166**, 1768 (1968); G. Veneziano, Nuovo. Cim **57A**, 190 (1968).
- [3] C. Lovelace, Phys. Lett. **34B**, 500 (1971).
- [4] *String theory in four dimensions* edited by M. Dine, North Holland, (1988).
- [5] I. Antoniadis, C. Buchas and C. Kounnas, Nucl. Phys. **B289**, 87 (1987).
- [6] D. Chang and A. Kumar, Phys. Rev. **D38**, 1893 (1988); *ibid* , **D38**, 3739 (1988).
- [7] H. Kawai, D. C. Lewellen and C-H. H. Tye, Nucl. Phys. **B288**, 1 (1987).
- [8] T. H. R. Skyrme, Proc. Roy. Soc. **A262**, 237 (1961).
- [9] M. B. Green, J. H. Schwarz and E. Witten, *Superstring Theory* Vol. 1, Cambridge University Press, Cambridge, England (1987), references to the materials presented in this work can be found in this book.
- [10] A. Neveu and J. H. Schwarz, Nucl Phys. **B31**, 86 (1971); A. Neveu, J. H. Schwarz and C. B. Thorn, *ibid* **35B**, 529 (1971).
- [11] M. A. Virasoro, Phys. Rev. **D1**, 2933 (1970).
- [12] L. D. Faddeev and V. N. Popov, Phys. Lett **25B**, 29 (1967); A. M. Polyakov, Phys. Lett. **103B**, 207 (1981), *ibid* **103B**, 211 (1981).
- [13] C. Becchi, A. Rouet and R. Stora, Phys. Lett. **52B**, 344 (1974); Ann. Phys. **98**, 287 (1976); I. V. Tyupin, Lebedev Preprint FIAN No. **39** (1975).
- [14] P. Ramond, Phys. Rev. **D3**, 2415 (1971).
- [15] D. Friedman, E. Martinec and S. Shanker, Nucl. Phys. **B271**, 93 (1986).
- [16] F. Gliozzi, J. Scherk and D. Olive, Phys. Lett. **65B**, 282 (1976) and references there in ; Nucl. Phys. **B122**, 253 (1977).
- [17] M. Kaku in *Strings, Conformal Fields and Topology* , Sringer-Verlag, New York(1991).
- [18] N. Seiberg and E. Witten, Nucl. Phys. **B276**, 272 (1986).
- [19] A. Erdelyi et al., *Higher Transcendental Functions*, Mc Graw Hill, New York, (1953).
- [20] *see for instance*, the introduction given in Superstring Construction, edited by B. Schellekens (North Holland, 1989).
- [21] R. Slansky, Phys. Rep. **79**, 1 (1981).
- [22] B. B. Deo, Mod. Phys. Lett. **A 13**, 2971 (1998).
- [23] J. C. Pati and A. Salam, Phys. Rev. **D10**, 275 (1979), R. N. Mohapatra and J. C. Pati, Phys. Rev. **D11**, 2558 (1975).
- [24] R. N. Mohapatra, Unification and Supersymmetry, Springer-Verlag, New York (1986); P. Langacker and S. Uma Sankar, Phys. Rev. **D40**, 1385 (1989).
- [25] F. D. Aguila and L. E. Ibanez, Nucl. Phys. **B177**, 60 (1981), C. S. Aulakh and R. N. Mohapatra Phys. Rev. **D28**, 217 (1983).
- [26] P. Langacker and N. Polonsky, Phys. Rev. **D 52**, 3081 (1995).
- [27] J. C. Pati, Intl. Jr. Mod. Phys. **A 14**, 2949 (1999).
- [28] Super-Kamiokanda Collaboration, Y. Fukuda et. al., Phys. Rev. Lett. **81**, 1562 (1998).